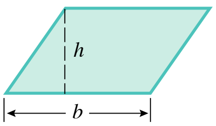
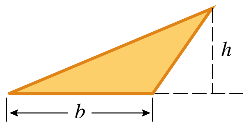
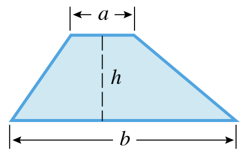
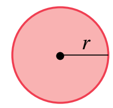
# GEOMETRY FORMULAS

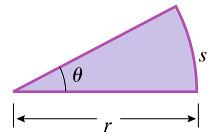
$A$ = area, $S$ = lateral surface area, $V$ = volume, $h$ = height, $B$ = area of base, $r$ = radius, $l$ = slant height, $C$ = circumference, $s$ = arc length

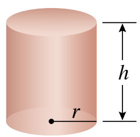
<table>
<tr><th>Parallelogram</th><th>Triangle</th><th>Trapezoid</th><th>Circle</th><th>Sector</th></tr>
<tr>
<td>

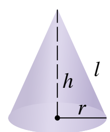
$A = bh$</td>
<td>

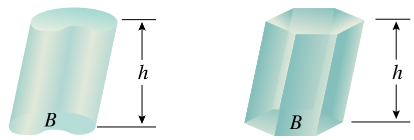
$A = \frac{1}{2}bh$</td>
<td>

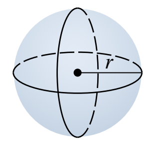
$A = \frac{1}{2}(a+b)h$</td>
<td>

$A = \pi r^2,\ C = 2\pi r$</td>
<td>

$A = \frac{1}{2}r^2\theta,\ s = r\theta$<br>($\theta$ in radians)</td>
</tr>
<tr><th>Right Circular Cylinder</th><th>Right Circular Cone</th><th colspan="2">Any Cylinder or Prism with Parallel Bases</th><th>Sphere</th></tr>
<tr>
<td>

$V = \pi r^2 h,\ S = 2\pi rh$</td>
<td>

$V = \frac{1}{3}\pi r^2 h,\ S = \pi rl$</td>
<td colspan="2">

$V = Bh$</td>
<td>

$V = \frac{4}{3}\pi r^3,\ S = 4\pi r^2$</td>
</tr>
</table>

# ALGEBRA FORMULAS

| THE QUADRATIC FORMULA | THE BINOMIAL FORMULA |
|---|---|
| The solutions of the quadratic equation $ax^2+bx+c=0$ are $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$ | $(x+y)^n = x^n + nx^{n-1}y + \dfrac{n(n-1)}{1\cdot 2}x^{n-2}y^2 + \dfrac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots + nxy^{n-1} + y^n$ <br> $(x-y)^n = x^n - nx^{n-1}y + \dfrac{n(n-1)}{1\cdot 2}x^{n-2}y^2 - \dfrac{n(n-1)(n-2)}{1\cdot 2\cdot 3}x^{n-3}y^3 + \cdots \pm nxy^{n-1} \mp y^n$ |

# TABLE OF INTEGRALS

## BASIC FUNCTIONS

1. $\displaystyle\int u^n\,du = \frac{u^{n+1}}{n+1} + C$
2. $\displaystyle\int \frac{du}{u} = \ln|u| + C$
3. $\displaystyle\int e^u\,du = e^u + C$
4. $\displaystyle\int \sin u\,du = -\cos u + C$
5. $\displaystyle\int \cos u\,du = \sin u + C$
6. $\displaystyle\int \tan u\,du = \ln|\sec u| + C$
7. $\displaystyle\int \sin^{-1} u\,du = u\sin^{-1} u + \sqrt{1-u^2} + C$
8. $\displaystyle\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$
9. $\displaystyle\int \tan^{-1} u\,du = u\tan^{-1} u - \ln\sqrt{1+u^2} + C$
10. $\displaystyle\int a^u\,du = \frac{a^u}{\ln a} + C$
11. $\displaystyle\int \ln u\,du = u\ln u - u + C$
12. $\displaystyle\int \cot u\,du = \ln|\sin u| + C$
13. $\displaystyle\int \sec u\,du = \ln|\sec u + \tan u| + C = \ln\left|\tan\left(\tfrac{1}{4}\pi + \tfrac{1}{2}u\right)\right| + C$
14. $\displaystyle\int \csc u\,du = \ln|\csc u - \cot u| + C = \ln\left|\tan \tfrac{1}{2}u\right| + C$
15. $\displaystyle\int \cot^{-1} u\,du = u\cot^{-1} u + \ln\sqrt{1+u^2} + C$
16. $\displaystyle\int \sec^{-1} u\,du = u\sec^{-1} u - \ln|u + \sqrt{u^2-1}| + C$
17. $\displaystyle\int \csc^{-1} u\,du = u\csc^{-1} u + \ln|u + \sqrt{u^2-1}| + C$

## RECIPROCALS OF BASIC FUNCTIONS

**18.** $\displaystyle\int \frac{1}{1 \pm \sin u}\,du = \tan u \mp \sec u + C$

**19.** $\displaystyle\int \frac{1}{1 \pm \cos u}\,du = -\cot u \pm \csc u + C$

**20.** $\displaystyle\int \frac{1}{1 \pm \tan u}\,du = \tfrac{1}{2}(u \pm \ln|\cos u \pm \sin u|) + C$

**21.** $\displaystyle\int \frac{1}{\sin u \cos u}\,du = \ln|\tan u| + C$

**22.** $\displaystyle\int \frac{1}{1 \pm \cot u}\,du = \tfrac{1}{2}(u \mp \ln|\sin u \pm \cos u|) + C$

**23.** $\displaystyle\int \frac{1}{1 \pm \sec u}\,du = u + \cot u \mp \csc u + C$

**24.** $\displaystyle\int \frac{1}{1 \pm \csc u}\,du = u - \tan u \pm \sec u + C$

**25.** $\displaystyle\int \frac{1}{1 \pm e^u}\,du = u - \ln(1 \pm e^u) + C$

## POWERS OF TRIGONOMETRIC FUNCTIONS

**26.** $\displaystyle\int \sin^2 u\,du = \tfrac{1}{2}u - \tfrac{1}{4}\sin 2u + C$

**27.** $\displaystyle\int \cos^2 u\,du = \tfrac{1}{2}u + \tfrac{1}{4}\sin 2u + C$

**28.** $\displaystyle\int \tan^2 u\,du = \tan u - u + C$

**29.** $\displaystyle\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$

**30.** $\displaystyle\int \cos^n u\,du = \frac{1}{n}\cos^{n-1} u\sin u + \frac{n-1}{n}\int \cos^{n-2} u\,du$

**31.** $\displaystyle\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$

**32.** $\displaystyle\int \cot^2 u\,du = -\cot u - u + C$

**33.** $\displaystyle\int \sec^2 u\,du = \tan u + C$

**34.** $\displaystyle\int \csc^2 u\,du = -\cot u + C$

**35.** $\displaystyle\int \cot^n u\,du = -\frac{1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$

**36.** $\displaystyle\int \sec^n u\,du = \frac{1}{n-1}\sec^{n-2} u\tan u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

**37.** $\displaystyle\int \csc^n u\,du = -\frac{1}{n-1}\csc^{n-2} u\cot u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$

## PRODUCTS OF TRIGONOMETRIC FUNCTIONS

**38.** $\displaystyle\int \sin mu \sin nu\,du = -\frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

**39.** $\displaystyle\int \cos mu \cos nu\,du = \frac{\sin(m+n)u}{2(m+n)} + \frac{\sin(m-n)u}{2(m-n)} + C$

**40.** $\displaystyle\int \sin mu \cos nu\,du = -\frac{\cos(m+n)u}{2(m+n)} - \frac{\cos(m-n)u}{2(m-n)} + C$

**41.** $\displaystyle\int \sin^m u \cos^n u\,du = -\frac{\sin^{m-1} u\cos^{n+1} u}{m+n} + \frac{m-1}{m+n}\int \sin^{m-2} u\cos^n u\,du$

$$= \frac{\sin^{m+1} u\cos^{n-1} u}{m+n} + \frac{n-1}{m+n}\int \sin^m u\cos^{n-2} u\,du$$

## PRODUCTS OF TRIGONOMETRIC AND EXPONENTIAL FUNCTIONS

**42.** $\displaystyle\int e^{au}\sin bu\,du = \frac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

**43.** $\displaystyle\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

## POWERS OF *u* MULTIPLYING OR DIVIDING BASIC FUNCTIONS

**44.** $\displaystyle\int u\sin u\,du = \sin u - u\cos u + C$

**45.** $\displaystyle\int u\cos u\,du = \cos u + u\sin u + C$

**46.** $\displaystyle\int u^2\sin u\,du = 2u\sin u + (2-u^2)\cos u + C$

**47.** $\displaystyle\int u^2\cos u\,du = 2u\cos u + (u^2-2)\sin u + C$

**48.** $\displaystyle\int u^n\sin u\,du = -u^n\cos u + n\int u^{n-1}\cos u\,du$

**49.** $\displaystyle\int u^n\cos u\,du = u^n\sin u - n\int u^{n-1}\sin u\,du$

**50.** $\displaystyle\int u^n\ln u\,du = \frac{u^{n+1}}{(n+1)^2}[(n+1)\ln u - 1] + C$

**51.** $\displaystyle\int ue^u\,du = e^u(u-1) + C$

**52.** $\displaystyle\int u^n e^u\,du = u^n e^u - n\int u^{n-1}e^u\,du$

**53.** $\displaystyle\int u^n a^u\,du = \frac{u^n a^u}{\ln a} - \frac{n}{\ln a}\int u^{n-1}a^u\,du + C$

**54.** $\displaystyle\int \frac{e^u\,du}{u^n} = -\frac{e^u}{(n-1)u^{n-1}} + \frac{1}{n-1}\int \frac{e^u\,du}{u^{n-1}}$

**55.** $\displaystyle\int \frac{a^u\,du}{u^n} = -\frac{a^u}{(n-1)u^{n-1}} + \frac{\ln a}{n-1}\int \frac{a^u\,du}{u^{n-1}}$

**56.** $\displaystyle\int \frac{du}{u\ln u} = \ln|\ln u| + C$

## POLYNOMIALS MULTIPLYING BASIC FUNCTIONS

**57.** $\displaystyle\int p(u)e^{au}\,du = \frac{1}{a}p(u)e^{au} - \frac{1}{a^2}p'(u)e^{au} + \frac{1}{a^3}p''(u)e^{au} - \cdots$ [signs alternate: $+ - + - \cdots$]

**58.** $\displaystyle\int p(u)\sin au\,du = -\frac{1}{a}p(u)\cos au + \frac{1}{a^2}p'(u)\sin au + \frac{1}{a^3}p''(u)\cos au - \cdots$ [signs alternate in pairs after first term: $+ + - - + + - - \cdots$]

**59.** $\displaystyle\int p(u)\cos au\,du = \frac{1}{a}p(u)\sin au + \frac{1}{a^2}p'(u)\cos au - \frac{1}{a^3}p''(u)\sin au - \cdots$ [signs alternate in pairs: $+ + - - + + - - \cdots$]

## RATIONAL FUNCTIONS CONTAINING POWERS OF $a + bu$ IN THE DENOMINATOR

**60.** $\displaystyle\int \frac{u\,du}{a+bu} = \frac{1}{b^2}[bu - a\ln|a+bu|] + C$

**61.** $\displaystyle\int \frac{u^2\,du}{a+bu} = \frac{1}{b^3}\left[\frac{1}{2}(a+bu)^2 - 2a(a+bu) + a^2\ln|a+bu|\right] + C$

**62.** $\displaystyle\int \frac{u\,du}{(a+bu)^2} = \frac{1}{b^2}\left[\frac{a}{a+bu} + \ln|a+bu|\right] + C$

**63.** $\displaystyle\int \frac{u^2\,du}{(a+bu)^2} = \frac{1}{b^3}\left[bu - \frac{a^2}{a+bu} - 2a\ln|a+bu|\right] + C$

**64.** $\displaystyle\int \frac{u\,du}{(a+bu)^3} = \frac{1}{b^2}\left[\frac{a}{2(a+bu)^2} - \frac{1}{a+bu}\right] + C$

**65.** $\displaystyle\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$

**66.** $\displaystyle\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$

**67.** $\displaystyle\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} + \frac{1}{a^2}\ln\left|\frac{u}{a+bu}\right| + C$

## RATIONAL FUNCTIONS CONTAINING $a^2 \pm u^2$ IN THE DENOMINATOR ($a > 0$)

**68.** $\displaystyle\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

**69.** $\displaystyle\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$

**70.** $\displaystyle\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$

**71.** $\displaystyle\int \frac{bu+c}{a^2+u^2}\,du = \frac{b}{2}\ln(a^2+u^2) + \frac{c}{a}\tan^{-1}\frac{u}{a} + C$

## INTEGRALS OF $\sqrt{a^2+u^2}$, $\sqrt{a^2-u^2}$, $\sqrt{u^2-a^2}$ AND THEIR RECIPROCALS ($a > 0$)

**72.** $\displaystyle\int \sqrt{u^2+a^2}\,du = \frac{u}{2}\sqrt{u^2+a^2} + \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

**73.** $\displaystyle\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$

**74.** $\displaystyle\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

**75.** $\displaystyle\int \frac{du}{\sqrt{u^2+a^2}} = \ln(u+\sqrt{u^2+a^2}) + C$

**76.** $\displaystyle\int \frac{du}{\sqrt{u^2-a^2}} = \ln|u+\sqrt{u^2-a^2}| + C$

**77.** $\displaystyle\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\frac{u}{a} + C$

## POWERS OF $u$ MULTIPLYING OR DIVIDING $\sqrt{a^2-u^2}$ OR ITS RECIPROCAL

**78.** $\displaystyle\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\sin^{-1}\frac{u}{a} + C$

**79.** $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u} = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

**80.** $\displaystyle\int \frac{\sqrt{a^2-u^2}\,du}{u^2} = -\frac{\sqrt{a^2-u^2}}{u} - \sin^{-1}\frac{u}{a} + C$

**81.** $\displaystyle\int \frac{u^2\,du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\sin^{-1}\frac{u}{a} + C$

**82.** $\displaystyle\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$

**83.** $\displaystyle\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{\sqrt{a^2-u^2}}{a^2u} + C$

## POWERS OF $u$ MULTIPLYING OR DIVIDING $\sqrt{u^2 \pm a^2}$ OR THEIR RECIPROCALS

**84.** $\displaystyle\int u\sqrt{u^2+a^2}\,du = \frac{1}{3}(u^2+a^2)^{3/2} + C$

**85.** $\displaystyle\int u\sqrt{u^2-a^2}\,du = \frac{1}{3}(u^2-a^2)^{3/2} + C$

**86.** $\displaystyle\int \frac{du}{u\sqrt{u^2+a^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

**87.** $\displaystyle\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C$

**88.** $\displaystyle\int \frac{\sqrt{u^2-a^2}\,du}{u} = \sqrt{u^2-a^2} - a\sec^{-1}\left|\frac{u}{a}\right| + C$

**89.** $\displaystyle\int \frac{\sqrt{u^2+a^2}\,du}{u} = \sqrt{u^2+a^2} - a\ln\left|\frac{a+\sqrt{u^2+a^2}}{u}\right| + C$

**90.** $\displaystyle\int \frac{du}{u^2\sqrt{u^2\pm a^2}} = \mp\frac{\sqrt{u^2\pm a^2}}{a^2u} + C$

**91.** $\displaystyle\int u^2\sqrt{u^2+a^2}\,du = \frac{u}{8}(2u^2+a^2)\sqrt{u^2+a^2} - \frac{a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

**92.** $\displaystyle\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$

**93.** $\displaystyle\int \frac{\sqrt{u^2+a^2}}{u^2}\,du = -\frac{\sqrt{u^2+a^2}}{u} + \ln(u+\sqrt{u^2+a^2}) + C$

**94.** $\displaystyle\int \frac{\sqrt{u^2-a^2}}{u^2}\,du = -\frac{\sqrt{u^2-a^2}}{u} + \ln|u+\sqrt{u^2-a^2}| + C$

**95.** $\displaystyle\int \frac{u^2}{\sqrt{u^2+a^2}}\,du = \frac{u}{2}\sqrt{u^2+a^2} - \frac{a^2}{2}\ln(u+\sqrt{u^2+a^2}) + C$

**96.** $\displaystyle\int \frac{u^2}{\sqrt{u^2-a^2}}\,du = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln|u+\sqrt{u^2-a^2}| + C$

## INTEGRALS CONTAINING $(a^2+u^2)^{3/2}$, $(a^2-u^2)^{3/2}$, $(u^2-a^2)^{3/2}$ ($a > 0$)

**97.** $\displaystyle\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$

**98.** $\displaystyle\int \frac{du}{(u^2\pm a^2)^{3/2}} = \pm\frac{u}{a^2\sqrt{u^2\pm a^2}} + C$

**99.** $\displaystyle\int (a^2-u^2)^{3/2}\,du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\sin^{-1}\frac{u}{a} + C$

**100.** $\displaystyle\int (u^2+a^2)^{3/2}\,du = \frac{u}{8}(2u^2+5a^2)\sqrt{u^2+a^2} + \frac{3a^4}{8}\ln(u+\sqrt{u^2+a^2}) + C$

**101.** $\displaystyle\int (u^2-a^2)^{3/2}\,du = \frac{u}{8}(2u^2-5a^2)\sqrt{u^2-a^2} + \frac{3a^4}{8}\ln|u+\sqrt{u^2-a^2}| + C$

## POWERS OF $u$ MULTIPLYING OR DIVIDING $\sqrt{a+bu}$ OR ITS RECIPROCAL

**102.** $\displaystyle\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2}+C$

**103.** $\displaystyle\int u^2\sqrt{a+bu}\,du = \frac{2}{105b^3}(15b^2u^2-12abu+8a^2)(a+bu)^{3/2}+C$

**104.** $\displaystyle\int u^n\sqrt{a+bu}\,du = \frac{2u^n(a+bu)^{3/2}}{b(2n+3)} - \frac{2an}{b(2n+3)}\int u^{n-1}\sqrt{a+bu}\,du$

**105.** $\displaystyle\int \frac{u\,du}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}+C$

**106.** $\displaystyle\int \frac{u^2\,du}{\sqrt{a+bu}} = \frac{2}{15b^3}(3b^2u^2-4abu+8a^2)\sqrt{a+bu}+C$

**107.** $\displaystyle\int \frac{u^n\,du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2an}{b(2n+1)}\int \frac{u^{n-1}\,du}{\sqrt{a+bu}}$

**108.** $\displaystyle\int \frac{du}{u\sqrt{a+bu}} = \begin{cases} \dfrac{1}{\sqrt{a}}\ln\left|\dfrac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right|+C & (a>0) \\[2ex] \dfrac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\dfrac{a+bu}{-a}}+C & (a<0) \end{cases}$

**109.** $\displaystyle\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int \frac{du}{u^{n-1}\sqrt{a+bu}}$

**110.** $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u} = 2\sqrt{a+bu}+a\int\frac{du}{u\sqrt{a+bu}}$

**111.** $\displaystyle\int \frac{\sqrt{a+bu}\,du}{u^n} = -\frac{(a+bu)^{3/2}}{a(n-1)u^{n-1}} - \frac{b(2n-5)}{2a(n-1)}\int\frac{\sqrt{a+bu}\,du}{u^{n-1}}$

## POWERS OF $u$ MULTIPLYING OR DIVIDING $\sqrt{2au-u^2}$ OR ITS RECIPROCAL

**112.** $\displaystyle\int \sqrt{2au-u^2}\,du = \frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

**113.** $\displaystyle\int u\sqrt{2au-u^2}\,du = \frac{2u^2-au-3a^2}{6}\sqrt{2au-u^2}+\frac{a^3}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

**114.** $\displaystyle\int \frac{\sqrt{2au-u^2}\,du}{u} = \sqrt{2au-u^2}+a\sin^{-1}\left(\frac{u-a}{a}\right)+C$

**115.** $\displaystyle\int \frac{\sqrt{2au-u^2}\,du}{u^2} = -\frac{2\sqrt{2au-u^2}}{u}-\sin^{-1}\left(\frac{u-a}{a}\right)+C$

**116.** $\displaystyle\int \frac{du}{\sqrt{2au-u^2}} = \sin^{-1}\left(\frac{u-a}{a}\right)+C$

**117.** $\displaystyle\int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au}+C$

**118.** $\displaystyle\int \frac{u\,du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2}+a\sin^{-1}\left(\frac{u-a}{a}\right)+C$

**119.** $\displaystyle\int \frac{u^2\,du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2}\sqrt{2au-u^2}+\frac{3a^2}{2}\sin^{-1}\left(\frac{u-a}{a}\right)+C$

## INTEGRALS CONTAINING $(2au-u^2)^{3/2}$

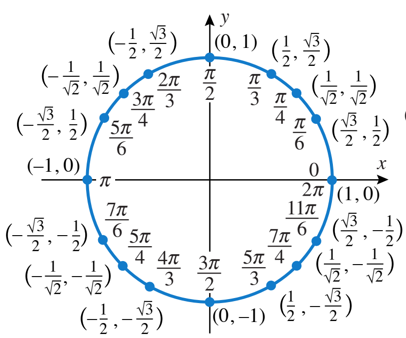
**120.** $\displaystyle\int \frac{du}{(2au-u^2)^{3/2}} = \frac{u-a}{a^2\sqrt{2au-u^2}}+C$

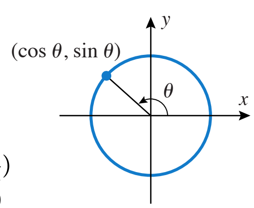
**121.** $\displaystyle\int \frac{u\,du}{(2au-u^2)^{3/2}} = \frac{u}{a\sqrt{2au-u^2}}+C$

## THE WALLIS FORMULA

**122.** $\displaystyle\int_0^{\pi/2}\sin^n u\,du = \int_0^{\pi/2}\cos^n u\,du = \frac{1\cdot 3\cdot 5\cdot\cdots\cdot(n-1)}{2\cdot 4\cdot 6\cdot\cdots\cdot n}\cdot\frac{\pi}{2}\ \left(\begin{array}{l} n \text{ an even} \\ \text{integer and} \\ n\ge 2\end{array}\right)$ or $\displaystyle\frac{2\cdot 4\cdot 6\cdot\cdots\cdot(n-1)}{3\cdot 5\cdot 7\cdot\cdots\cdot n}\ \left(\begin{array}{l} n \text{ an odd} \\ \text{integer and} \\ n\ge 3\end{array}\right)$

# TRIGONOMETRY REVIEW




## PYTHAGOREAN IDENTITIES

$\sin^2\theta+\cos^2\theta=1$ $\qquad \tan^2\theta+1=\sec^2\theta$ $\qquad 1+\cot^2\theta=\csc^2\theta$

## SIGN IDENTITIES

$\sin(-\theta)=-\sin\theta$ $\qquad \cos(-\theta)=\cos\theta$ $\qquad \tan(-\theta)=-\tan\theta$

$\csc(-\theta)=-\csc\theta$ $\qquad \sec(-\theta)=\sec\theta$ $\qquad \cot(-\theta)=-\cot\theta$

## COMPLEMENT IDENTITIES

$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$ $\qquad \cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$ $\qquad \tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$

$\csc\left(\frac{\pi}{2}-\theta\right)=\sec\theta$ $\qquad \sec\left(\frac{\pi}{2}-\theta\right)=\csc\theta$ $\qquad \cot\left(\frac{\pi}{2}-\theta\right)=\tan\theta$

## SUPPLEMENT IDENTITIES

$\sin(\pi-\theta)=\sin\theta$ $\qquad \cos(\pi-\theta)=-\cos\theta$ $\qquad \tan(\pi-\theta)=-\tan\theta$

$\csc(\pi-\theta)=\csc\theta$ $\qquad \sec(\pi-\theta)=-\sec\theta$ $\qquad \cot(\pi-\theta)=-\cot\theta$

$\sin(\pi+\theta)=-\sin\theta$ $\qquad \cos(\pi+\theta)=-\cos\theta$ $\qquad \tan(\pi+\theta)=\tan\theta$

$\csc(\pi+\theta)=-\csc\theta$ $\qquad \sec(\pi+\theta)=-\sec\theta$ $\qquad \cot(\pi+\theta)=\cot\theta$

## ADDITION FORMULAS

$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

$\displaystyle\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

$\displaystyle\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

## DOUBLE-ANGLE FORMULAS

$\sin 2\alpha=2\sin\alpha\cos\alpha$ $\qquad \cos 2\alpha=2\cos^2\alpha-1$

$\cos 2\alpha=\cos^2\alpha-\sin^2\alpha$ $\qquad \cos 2\alpha=1-2\sin^2\alpha$

## HALF-ANGLE FORMULAS

$\displaystyle\sin^2\frac{\alpha}{2}=\frac{1-\cos\alpha}{2}$ $\qquad \displaystyle\cos^2\frac{\alpha}{2}=\frac{1+\cos\alpha}{2}$