Derivative Problems

Find the derivative of each of the given functions.

1. $y = (x^2 + 4x + 6)^5$

2. $f(t) = \dfrac{1}{(t^2 - 2t - 5)^4}$

3. $h(t) = \left(t - \dfrac{1}{t}\right)^{3/2}$

4. $y = \dfrac{1}{\sqrt[5]{x^2}}$

5. $G(x) = (3x - 2)^{10}(5x^2 - x + 1)^{12}$

6. $y = (2x - 4)^4(8x^2 - 4)^{-3}$

7. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$

8. $y = \sec^2 x + \tan^2 x$

9. $R(y) = \dfrac{y^2 - 1}{(3y + 1)^2}$

10. $f(x) = \left(3x^2\right)(4x)^{1/2}$

11. $g(x) = \dfrac{2}{x^4 - x^2 + 1}$

12. $h(x) = \sqrt[5]{(3x^2 - 2x)^4}$

13. $f(x) = (\sin x \ \sin 3x)^9$

14. $y = \dfrac{1 + \sin x}{x + \cos x}$

15. $f(x) = \dfrac{\cos x}{\sin x}(\sin x + \tan x)$

16. $g(x) = \dfrac{2x^4 + 3x^2 - 1}{x^2}$

17. $s(t) = \sqrt[4]{\dfrac{t^3 + 1}{t^3 - 1}}$

18. $f(z) = \dfrac{1}{\sqrt[5]{2z - 1}}$

19. $h(x) = \dfrac{x}{\sqrt{7 - 3x}}$

20. $f(x) = \left(2x^{3/4} + 5x^{-1/6}\right)^{12}$

21. $y = \dfrac{\cos(a^3 + x^3)}{3}$

22. $s = \left(\dfrac{1 + t^2}{1 - t^2}\right)^7$

23. $y = \dfrac{\tan x - 1}{\sec x}$

24. $f(x) = \dfrac{\tan^2 x}{\sqrt{\sin^6 x + \sin^4 x \cos^2 x}}$

25. $h(x) = \left(x^2 + (x^2 + 9)^{1/2}\right)^{1/2}$

26. $r(t) = \sqrt[3]{\dfrac{2t + 5}{7t - 2}}$

27. $y = \sqrt{x + \sqrt{x}}$

28. $R = \dfrac{\sqrt{t} + 1}{\sqrt{t} - 1}$

29. $f(x) = \left(\dfrac{\cos(x^2)\ \tan^2(x^2)}{\sec(x^2)}\right)^3$

30. $y = \dfrac{2x}{(3x^2 - 4)^{1/3}}$

Find the **second** derivative of each of the given functions.

31. $g(x) = \dfrac{2x + 1}{x - 1}$

32. $f(y) = \dfrac{y}{\sqrt{1 - y^2}}$

33. $h(x) = \dfrac{3x}{\sqrt{2x^2 + 7}}$

34. $y = \dfrac{3}{(5 - 2x^2)^{3/4}}$

35. If f and g are differentiable functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = -5$, and $g'(2) = 2$, find the following values.

(a) $(f + g)'(2)$

(b) $(4f)'(2)$

(c) $(fg)'(2)$

(d) $(ff)'(2)$

(e) $\left(\frac{1}{f+g}\right)'(2)$

(f) $\left(\frac{5}{g}\right)'(2)$

36. Given the following table of values, find the indicated derivatives.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	2	6
2	1	5	8	7
3	7	7	2	9

(a) $h'(1)$, where $h(x) = f(g(x))$

(b) $H'(1)$, where $H(x) = g(f(x))$

(c) $F'(2)$, where $F(x) = f(f(x))$

(d) $G'(3)$, where $G(x) = g(g(x))$

Solutions:

1. $y' = 10(x^2+4x+6)^4(x+2)$

2. $f'(t) = \dfrac{8(1-t)}{(t^2-2t-5)^5}$

3. $h'(t) = \dfrac{3}{2}\left(t-\dfrac{1}{t}\right)^{1/2}\left(1+\dfrac{1}{t^2}\right)$

4. $y' = \dfrac{-2}{5\sqrt[5]{x^7}}$

5. $G'(x) = 30(3x-2)^9(5x^2-x+1)^{12} + 12(10x-1)(3x-2)^{10}(5x^2-x+1)^{11}$
$= 6(3x-2)^9(5x^2-x+1)^{11}(85x^2-51x+9)$

6. $y' = 8(2x-4)^3(8x^2-4)^{-3} - 48x(2x-4)^4(8x^2-4)^{-4}$

7. $y' = 2x\sqrt[3]{x^2+2} + \left(\dfrac{2x}{3}\right)(x^2+1)(x^2+2)^{-2/3}$

8. $y' = 4\sec^2 x \tan x$

9. $R'(y) = \dfrac{2(y+3)}{(3y+1)^3}$

10. $f'(x) = 15x^{3/2}$

11. $g'(x) = -2(x^4-x^2+1)^{-2}(4x^3-2x)$

12. $h'(x) = \dfrac{8(3x-1)}{5(3x^2-2x)^{1/5}}$

13. $f'(x) = 9(\sin x\ \sin 3x)^8(\cos x \sin 3x + 3\sin x \cos 3x)$

14. $y' = \dfrac{x\cos x}{(x+\cos x)^2}$

15. $f'(x) = -\sin x$

16. $g'(x) = 4x + 2x^{-3}$

17. $s'(t) = -\dfrac{1}{2}\left(\dfrac{t^3+1}{t^3-1}\right)^{-3/4}\dfrac{3t^2}{(t^3-1)^2}$

18. $f'(z) = -\dfrac{2}{5}(2z-1)^{-6/5}$

19. $h'(x) = \dfrac{14-3x}{2(7-3x)^{3/2}}$

20. $f'(x) = 12\left(2x^{3/4}+5x^{-1/6}\right)^{11}\left(\dfrac{3}{2}x^{-1/4} - \dfrac{5}{6}x^{-7/6}\right)$

21. $y' = -x^2\sin(a^3+x^3)$

22. $\dfrac{ds}{dt} = 7\left(\dfrac{1+t^2}{1-t^2}\right)^6\dfrac{4t}{(1-t^2)^2}$

23. $y = \cos x + \sin x$

24. $f'(x) = 2\sec^2 x \tan x$

25. $h'(x) = \dfrac{1}{2}\left(x^2+(x^2+9)^{1/2}\right)^{-1/2}(2x + x(x^2+9)^{-1/2})$

26. $r'(t) = \left(\dfrac{2t+5}{7t-2}\right)^{-2/3}\dfrac{-13}{(7t-2)^2}$

27. $y' = \dfrac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\dfrac{1}{2\sqrt{x}}\right)$

28. $\dfrac{dR}{dt} = \dfrac{-1}{\sqrt{t}(\sqrt{t}-1)^2}$

29. $f'(x) = 12x\ \sin^5(x^2)\cos(x^2)$

30. $y' = \dfrac{2x^2-8}{(3x^2-4)^{4/3}}$

31. $g''(x) = 6(x-1)^{-3}$

32. $f''(y) = 3y(1-y^2)^{-5/2}$

33. $h''(x) = -126x(2x^2+7)^{-5/2}$

34. $\dfrac{d^2y}{dx^2} = \dfrac{45(1+x^2)}{(5-2x^2)^{11/4}}$

35. (a) 1 (b) -4
(c) 11 (d) -6
(e) $-\frac{1}{4}$ (f) $-\frac{2}{5}$

36. (a) $h'(1) = 30$ (b) $H'(1) = 36$
(c) $F'(2) = 20$ (d) $G'(3) = 63$