

1 Fundamental Concepts From Algebra & Precalculus

1.1 Review Exercises

1.1.1 Simplify each expression.

$$1. \frac{5 \cdot 2 - (4 - 7)^2}{\left[\frac{1}{2}(3^2 - (2 - 5))\right]^2}$$

$$3. \frac{8 - 3[-2(2 - 5) - 4(8 - 6)]}{2[9^2 + 8 - 3 \cdot 5^2]}$$

$$5. \frac{-5 + 2\left[\left(-\frac{1}{4}\right)(16)\right]^2 - (4 - 7)^2}{7^2 + 6 - 4 \cdot 5^2}$$

$$7. 4\sqrt{12} - 2\sqrt{75}$$

$$9. 3x\sqrt[3]{54x} - 5\sqrt[3]{16x^4}$$

$$11. (8x^{-6}y^3)^{\frac{1}{3}}(x^{\frac{5}{6}}y^{-\frac{1}{3}})^6$$

$$13. \left(\frac{2x^{\frac{2}{3}}}{y^{\frac{1}{2}}}\right)^2 \left(\frac{3x^{-\frac{5}{6}}}{y^{\frac{1}{3}}}\right)$$

$$15. \sqrt[3]{24xy^3} - y\sqrt[3]{27x}\sqrt[3]{3}$$

$$17. \frac{(2^{-1}x^{-2}y^{-\frac{1}{3}})^{-4}(-4xy^{-\frac{1}{3}})^{-2}}{(8^{\frac{1}{2}}x^2y^{\frac{3}{2}})^2}$$

$$2. \frac{2\left|(-\frac{1}{3})(-15)\right| - (4 - 7)^2}{9^2 + 8 - 3 \cdot 5^2}$$

$$4. \frac{9^2 + 8 - 3 \cdot 5^2}{\left(-\frac{1}{2}\right)(-28) + 3 - 2|2^2 - 3^2|}$$

$$6. \frac{8 - 3(|-2(5 - 2)| - 4|6 - 8|)}{\left(-\frac{1}{3}\right)(-27) + 4 - 2|\sqrt{1} - \sqrt{16}|}$$

$$8. 3\sqrt[3]{24} + \sqrt[3]{81}$$

$$10. \left(125x^{\frac{9}{4}}\right)^{\frac{2}{3}} \left(\frac{15x^{\frac{3}{4}}}{5x^{\frac{1}{2}}}\right)$$

$$12. \left(\frac{x^{\frac{1}{2}}y^{-\frac{7}{4}}}{y^{-\frac{5}{4}}}\right)^{-4}$$

$$14. \left(\frac{-8x^3}{y^{-6}}\right)^{\frac{2}{3}} \left(\frac{-x^{-\frac{2}{3}}y^{\frac{3}{2}}}{y^{-\frac{1}{3}}}\right)^3$$

$$16. (2x^2y^{-5})(9x^{-6}y)^{1/2} \left(\frac{1}{3}x^{-1}y^{-1/4}\right)^2$$

$$18. (27x^{-6}y^2)^{\frac{2}{3}} \left(\frac{1}{2}x^{\frac{6}{5}}y^{-\frac{1}{15}}\right)^5$$

1.1.2 Factor the expression completely (and simplify).

$$19. 36y^4 + 24y^2$$

$$21. x^2 - 144$$

$$23. 81a^2 - b^6$$

$$25. 8z^3 - 27$$

$$27. 25x^2 + 35x + 10$$

$$29. y^3 - 6y^2 + 2y - 12$$

$$31. x^2 - 14x + 45$$

$$33. 6x^2 - 7xy - 5y^2$$

$$35. 64x^2 - 16x + 1$$

$$37. x^3 + 2x^2 - x - 2$$

$$39. x^3 + 3x^2 - 25x - 75$$

$$41. 6x^2y - \frac{27}{2}y - 2x^2 + \frac{9}{2}$$

$$43. (2x^2 - 3x + 1)(4)(3x + 2)^3(3) + (3x + 2)^4(4x - 3)$$

$$20. 9x^3y + 6xy^2$$

$$22. 121 - y^2$$

$$24. x^4 - 16$$

$$26. x^2 + 4x + 4$$

$$28. -6u^2 - 3u + 63$$

$$30. 4x^3 - 2x^2 + 6x - 3$$

$$32. 6x^2 - 17x + 12$$

$$34. 36x^2 - 49y^2$$

$$36. 5x^3 - 45x$$

$$38. \frac{5}{2}x^3 + 5x^2 - 10x - 20$$

$$40. 48y^4 - 3y^2$$

$$42. 12x^2(x - 1) - 4x(x - 1) - 5x + 5$$

$$44. (6x - 5)^3(2)(x^2 + 4)(2x) + (x^2 + 4)^2(3)(6x - 5)^2(6)$$

45. $-5(x^2 + 4)^4(2x)(x - 2)^4 + (x^2 + 4)^5(4)(x - 2)^3$

47. $3(2x - 1)^2(2)(x + 3)^{1/2} + (2x - 1)^3(\frac{1}{2})(x + 3)^{-1/2}$

49. $3x^2(2x - 3) + 3x(2x - 3) - 36x + 54$

51. $x^3 - 7x + 6$

46. $3(2x - 3)^2(2)(9 - 2x^3)^5 + (2x - 3)^3(5)(9 - 2x^3)^4(-6x^2)$

48. $(x^2 + 3)^{-1/3} - \frac{2}{3}x^2(x^2 + 3)^{-4/3}$

50. $(x + y)^4 - 100(x + y)^2$

52. $x^3 - 5x^2 - 2x + 24$

1.1.3 Complete the Square (without changing the value of the expression).

53. $x^2 + 6x$

55. $y^2 - 8y + 12$

57. $5x^2 - 3x - 8$

54. $t^2 - 9t$

56. $-x^2 + 2x + 24$

58. $-3x^2 + 3x + 6$

1.1.4 Perform the Indicated Operations (and completely simplify).

59. $25 + \frac{10}{x + 4}$

61. $\frac{3x}{x - 8} - \frac{6}{8 - x}$

63. $\frac{x}{x^2 - 9} + \frac{3}{x(x - 3)}$

65. $\frac{4}{x} - \frac{2}{x^2} + \frac{4}{x + 3}$

67. $\frac{1}{x^2 - 2x - 8} \div \left(\frac{1}{x - 4} - \frac{1}{x + 2} \right)$

69. $\frac{3 + \frac{9}{x - 3}}{4 + \frac{12}{x - 3}}$

71. $\frac{\frac{r}{s^2} + \frac{s}{r^2}}{\frac{s}{r^2} - \frac{r}{s^2}}$

73. $\frac{x^2 - 4}{x^2 - 4x + 4} \cdot \frac{6 - 3x}{x + 2}$

75. $\frac{4x^2 - 16}{x^2 + 3x - 10} \div \frac{2x^2 + 10x + 12}{x^2 + 8x + 15}$

77. $\frac{(x + h)^2 - 3(x + h) - (x^2 - 3x)}{h}$

79. $(3x + 2)^{\frac{1}{3}}(2)(4x - 5)(4) + (4x - 5)^2(\frac{1}{3})(3x + 2)^{-\frac{2}{3}}(3)$

81. $(x^2 + 9)^4(-\frac{1}{3})(x + 6)^{-\frac{4}{3}} + 4(x^2 + 9)^3(2x)(x + 6)^{-\frac{1}{3}}$

83. $\frac{3x^2(x^2 - 5)^4 - x^3(4)(x^2 - 5)^3(2x)}{[(x^2 - 5)^4]^2}$

85. $\frac{2x(1 - x^2)^{\frac{1}{2}} - x^2(\frac{1}{2})(1 - x^2)^{-\frac{1}{2}}(-2x)}{[(1 - x^2)^{\frac{1}{2}}]^2}$

60. $\frac{100}{x - 10} - 8$

62. $\frac{2y}{10y - 6} - \frac{3}{3 - 5y}$

64. $\frac{2x - 1}{x^2 - 3x - 10} + \frac{x}{x^2 - 4}$

66. $\frac{5}{2x + 2} - \frac{1}{2x} - \frac{3}{2(x + 1)^2}$

68. $\left(4 - \frac{3}{x + 2} \right) \left(1 + \frac{3}{x - 1} \right)$

70. $\frac{\frac{x + 1}{x + 2} - \frac{1}{x}}{\frac{2}{x + 2}}$

72. $\frac{\frac{x + h}{x + h + 1} - \frac{x}{x + 1}}{h}$

74. $\frac{9x^2 - 4}{\frac{9}{2}x^2 - 3x + 2} \cdot \frac{9x^4 - 6x^3 + 4x^2}{3x^3 - 5x^2 + 2x}$

76. $\frac{x^3 - 25x}{4x^2} \cdot \frac{2x^2 - 2}{x^2 - 6x + 5} \div \frac{\frac{1}{5}x^2 + x}{7x + 7}$

78. $\frac{x^{-1} - (x + 2)^{-1}}{2}$

80. $(3x + 1)^6(\frac{1}{2})(2x - 5)^{-\frac{1}{2}}(2) + 6(3x + 1)^5(3)(2x - 5)^{\frac{1}{2}}$

82. $\frac{(2x)(x^2 - 1)^4 - x^2(4)(x^2 - 1)^3(2x)}{[(x^2 - 1)^4]^2}$

84. $\frac{3(x^2 + 4)^{\frac{1}{3}} - 3x(\frac{1}{3})(x^2 + 4)^{-\frac{2}{3}}(2x)}{[(x^2 + 4)^{\frac{1}{3}}]^2}$

86. $\frac{(\frac{1}{3})(2x + 3)^{-\frac{2}{3}}(2)(3x + 2)^{\frac{1}{2}} - (2x + 3)^{\frac{1}{3}}(\frac{1}{2})(3x + 2)^{-\frac{1}{2}}(3)}{[(3x + 2)^{\frac{1}{2}}]^2}$

1.1.5 Rationalize the Numerator (and completely simplify).

87. $\frac{\sqrt{x} - \sqrt{y}}{x^2 - y^2}$

88. $\frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$

1.1.6 Solve the Equation.

89. $x^2 - x - 12 = 0$

90. $3x^2 + 5x = 12$

91. $2x - 4 = 2(x - 3)$

92. $3x + 2(x - 10) = 5(x - 4)$

93. $x^2 - 6x + 11 = 2$

94. $(x + 2)(x + 4) = 3$

95. $3x^3 = 15x^2 + 18x$

96. $x^3 + 84x = 19x^2$

97. $x^2 - 4x - 3 = 0$

98. $2x^2 + 8x + 3 = 0$

99. $2x^2 - 4x + 3 = 0$

100. $(x + 1)\left(\frac{2}{5}x - 1\right) = \frac{2}{5}$

101. $\left(\frac{1}{4}x - \frac{1}{2}\right)(3x + 3) = 3$

102. $\left(\frac{2}{3}x - 1\right)\left(\frac{1}{2}x + \frac{1}{2}\right) = \frac{1}{2}$

103. $z^2(z + 2) - 4(z + 2) = 0$

104. $x^4 + x^3 - 4x^2 - 4x = 0$

105. $\frac{x+1}{4} = \frac{1}{6} + \frac{2-x}{3}$

106. $\frac{3x}{5} - \frac{x-3}{2} = \frac{x+2}{3}$

107. $\frac{3}{x+4} - 7 = \frac{-4}{x+4}$

108. $\frac{15}{6t^2 - t - 1} + \frac{3}{2t - 1} = \frac{2}{3t + 1}$

109. $\frac{-3}{x+4} + \frac{7}{x-4} = \frac{-5x+4}{x^2-16}$

110. $S = \frac{p}{q + p(1 - q)}$ for q

111. $\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$ for q

112. $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$ for R_2

113. $\sqrt{20 - 8x} = x$

114. $\sqrt{3 - x} - x = 3$

115. $x = 3 + \sqrt{5x - 9}$

116. $1 + \sqrt{6x + 1} = x$

117. $x^3 - 17x + 4 = 0$ given that 4 is a root

118. $3x^3 + 7x^2 - 22x - 8 = 0$ given that $-\frac{1}{3}$ is a root

1.1.7 Solve the Absolute Value Inequality - write the solution as an interval.

119. $|3x + 5| < 17$

120. $\left|\frac{3(x-1)}{4}\right| < 6$

121. $\left|3 - \frac{3}{4}x\right| > 9$

122. $1 < \left|x - \frac{11}{3}\right| + \frac{7}{3}$

123. $-|2 - \frac{x}{2}| + 1 \geq -1$

124. $-4|x + 2| + 5 \leq -7$

1.1.8 Polynomial Division

125. $(x^4 - 10x^2 - 2x + 3) \div (x + 3)$

126. $(x^3 - 2) \div (x - 1)$

127. $(-13x^3 + 10x^4 + 8x - 7x^2 + 4) \div (3 - 2x)$

128. $(x^4 + 6x^3 + 6x^2 - 10x - 3) \div (2x - 3)$

129. $\frac{x^4 + 6x^3 + 6x^2 - 10x - 3}{x^2 + 2x - 3}$

130. $\frac{2x^3 + 2x^2 - 2x - 15}{2x^2 + 4x + 5}$

131. $\frac{2x^5 - 8x^4 + 2x^3 + x^2}{2x^3 + 1}$

132. $\frac{4x^4 + 6x^3 + 3x - 1}{2x^2 + 1}$

1.1.9 Function Notation - evaluate the function as indicated, and simplify.

133. $f(x) = 3 - 7x$

(a) $f(-1)$

(b) $f(\frac{1}{2})$

(c) $f(t) - f(-2)$

(d) $f(x + 1)$

134. $f(x) = \frac{3x}{x-5}$

(a) $f(0)$

(b) $f(\frac{5}{3})$

(c) $f(2) - f(-1)$

(d) $f(x + 4)$

135. $g(x) = \frac{x+2}{x+1}$

(a) $g(0)$

(b) $g(-1)$

(c) $g(3) + g(-5)$

(d) $f(x - 2)$

136. $f(x) = \begin{cases} x + 8, & x < 0 \\ 10 - 2x, & x \geq 0 \end{cases}$

(a) $f(4)$

(b) $f(-10)$

(c) $f(0)$

(d) $f(6) - f(-2)$

137. $f(x) = \begin{cases} 12, & x < -2 \\ 5x - 4, & -2 \leq x < 2 \\ 10x, & x \geq 2 \end{cases}$

(a) $f(-2)$

(b) $f(2)$

(c) $f(1) + f(3)$

(d) $f(-5) + f(-4)$

138. $f(x) = 2x^2 + 5$

(a) $f(x + 2) - f(2)$

(b) $f(x + 1) - f(x)$

(c) $\frac{f(x-3) - f(3)}{x}$

(d) $\frac{f(x+h) - f(x)}{h}$

§ In exercises 139-142, find and simplify the difference quotient $\frac{f(x+h) - f(x)}{h}$ for each function.

139. $f(x) = 3x^2 + x + 5$

140. $f(x) = -x^2 - 3x + 1$

141. $f(x) = \frac{1}{2x^2}$

142. $f(x) = \sqrt{x-1}$

1.1.10 Function Operations

§ In exercises 143-146, below find

(a) $(f \circ g)(x)$

(b) $(g \circ f)(x)$

(c) $(f \circ g)(3)$

143. $f(x) = x^2 + 3, g(x) = 4x - 1$

144. $f(x) = \sqrt{x}, g(x) = x + 1$

145. $f(x) = 6x - 3, g(x) = \frac{x+3}{6}$

146. $f(x) = 5x - 2, g(x) = -x^2 + 4x - 1$

§ In exercises 147-150,

(A) Find an equation for f^{-1} , the inverse function.

(B) Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

147. $f(x) = 4x - 3$

148. $f(x) = 8x^3 + 1$

149. $f(x) = \frac{2}{x} + 5$

150. $f(x) = \frac{2x-3}{x+1}$

§ Solve each inequality in exercises 151-154, and express each solution set in interval notation.

151. $\frac{3x+5}{6-2x} \geq 0$

152. $\frac{1}{x+1} \geq \frac{2}{x-1}$

153. $\frac{x^2-3x+2}{x^2-2x-3} \geq 0$

154. $\frac{x^4-3x^2-4}{x^2-x-20} < 0$