1 Fundamental Concepts From Algebra & Precalculus

1.1 Review Exercises

1.1.1 Simplify each expression.

1. $\dfrac{5 \cdot 2 - (4-7)^2}{\left[\frac{1}{2}(3^2 - (2-5))\right]^2}$

2. $\dfrac{2\left|\left(-\frac{1}{3}\right)(-15)\right| - (4-7)^2}{9^2 + 8 - 3 \cdot 5^2}$

3. $\dfrac{8 - 3[-2(2-5) - 4(8-6)]}{2[9^2 + 8 - 3 \cdot 5^2]}$

4. $\dfrac{9^2 + 8 - 3 \cdot 5^2}{\left(-\frac{1}{2}\right)(-28) + 3 - 2|2^2 - 3^2|}$

5. $\dfrac{-5 + 2\left[(-\frac{1}{4})(16)\right]^2 - (4-7)^2}{7^2 + 6 - 4 \cdot 5^2}$

6. $\dfrac{8 - 3\left(\,|-2(5-2)| - 4\,|6-8|\,\right)}{\left(-\frac{1}{3}\right)(-27) + 4 - 2\left|\sqrt{1} - \sqrt{16}\right|}$

7. $4\sqrt{12} - 2\sqrt{75}$

8. $3\sqrt[3]{24} + \sqrt[3]{81}$

9. $3x\sqrt[3]{54x} - 5\sqrt[3]{16x^4}$

10. $\left(125x^{\frac{9}{4}}\right)^{\frac{2}{3}}\left(\dfrac{15x^{\frac{3}{4}}}{5x^{\frac{1}{2}}}\right)$

11. $(8x^{-6}y^3)^{\frac{1}{3}}(x^{\frac{5}{6}}y^{-\frac{1}{3}})^6$

12. $\left(\dfrac{x^{\frac{1}{2}}y^{-\frac{7}{4}}}{y^{-\frac{5}{4}}}\right)^{-4}$

13. $\left(\dfrac{2x^{\frac{2}{3}}}{y^{\frac{1}{2}}}\right)^2\left(\dfrac{3x^{-\frac{5}{6}}}{y^{\frac{1}{3}}}\right)$

14. $\left(\dfrac{-8x^3}{y^{-6}}\right)^{\frac{2}{3}}\left(\dfrac{-x^{-\frac{2}{3}}y^{\frac{3}{2}}}{y^{-\frac{1}{3}}}\right)^3$

15. $\sqrt[3]{24xy^3} - y\sqrt[3]{27x}\sqrt[3]{3}$

16. $(2x^2y^{-5})(9x^{-6}y)^{1/2}\left(\dfrac{1}{3}x^{-1}y^{-1/4}\right)^2$

17. $\dfrac{(2^{-1}x^{-2}y^{-\frac{1}{3}})^{-4}(-4xy^{-\frac{1}{3}})^{-2}}{(8^{\frac{1}{2}}x^2y^{\frac{3}{2}})^2}$

18. $\left(27x^{-6}y^2\right)^{\frac{2}{3}}\left(\dfrac{1}{2}x^{\frac{6}{5}}y^{-\frac{1}{15}}\right)^5$

1.1.2 Factor the expression completely (and simplify).

19. $36y^4 + 24y^2$

20. $9x^3y + 6xy^2$

21. $x^2 - 144$

22. $121 - y^2$

23. $81a^2 - b^6$

24. $x^4 - 16$

25. $8z^3 - 27$

26. $x^2 + 4x + 4$

27. $25x^2 + 35x + 10$

28. $-6u^2 - 3u + 63$

29. $y^3 - 6y^2 + 2y - 12$

30. $4x^3 - 2x^2 + 6x - 3$

31. $x^2 - 14x + 45$

32. $6x^2 - 17x + 12$

33. $6x^2 - 7xy - 5y^2$

34. $36x^2 - 49y^2$

35. $64x^2 - 16x + 1$

36. $5x^3 - 45x$

37. $x^3 + 2x^2 - x - 2$

38. $\frac{5}{2}x^3 + 5x^2 - 10x - 20$

39. $x^3 + 3x^2 - 25x - 75$

40. $48y^4 - 3y^2$

41. $6x^2y - \frac{27}{2}y - 2x^2 + \frac{9}{2}$

42. $12x^2(x-1) - 4x(x-1) - 5x + 5$

43. $(2x^2 - 3x + 1)(4)(3x+2)^3(3) + (3x+2)^4(4x-3)$

44. $(6x-5)^3(2)(x^2+4)(2x) + (x^2+4)^2(3)(6x-5)^2(6)$

45. $-5(x^2+4)^4(2x)(x-2)^4 + (x^2+4)^5(4)(x-2)^3$

46. $3(2x-3)^2(2)(9-2x^3)^5 + (2x-3)^3(5)(9-2x^3)^4(-6x^2)$

47. $3(2x-1)^2(2)(x+3)^{1/2} + (2x-1)^3(\frac{1}{2})(x+3)^{-1/2}$

48. $(x^2+3)^{-1/3} - \frac{2}{3}x^2(x^2+3)^{-4/3}$

49. $3x^2(2x-3) + 3x(2x-3) - 36x + 54$

50. $(x+y)^4 - 100(x+y)^2$

51. $x^3 - 7x + 6$

52. $x^3 - 5x^2 - 2x + 24$

1.1.3 Complete the Square (without changing the value of the expression).

53. $x^2 + 6x$

54. $t^2 - 9t$

55. $y^2 - 8y + 12$

56. $-x^2 + 2x + 24$

57. $5x^2 - 3x - 8$

58. $-3x^2 + 3x + 6$

1.1.4 Perform the Indicated Operations (and completely simplify).

59. $25 + \dfrac{10}{x+4}$

60. $\dfrac{100}{x-10} - 8$

61. $\dfrac{3x}{x-8} - \dfrac{6}{8-x}$

62. $\dfrac{2y}{10y-6} - \dfrac{3}{3-5y}$

63. $\dfrac{x}{x^2-9} + \dfrac{3}{x(x-3)}$

64. $\dfrac{2x-1}{x^2-3x-10} + \dfrac{x}{x^2-4}$

65. $\dfrac{4}{x} - \dfrac{2}{x^2} + \dfrac{4}{x+3}$

66. $\dfrac{5}{2x+2} - \dfrac{1}{2x} - \dfrac{3}{2(x+1)^2}$

67. $\dfrac{1}{x^2-2x-8} \div \left(\dfrac{1}{x-4} - \dfrac{1}{x+2} \right)$

68. $\left(4 - \dfrac{3}{x+2} \right) \left(1 + \dfrac{3}{x-1} \right)$

69. $\dfrac{3 + \dfrac{9}{x-3}}{4 + \dfrac{12}{x-3}}$

70. $\dfrac{\dfrac{x+1}{x+2} - \dfrac{1}{x}}{\dfrac{2}{x+2}}$

71. $\dfrac{\dfrac{r}{s} + \dfrac{s}{r}}{\dfrac{r^2}{s^2} - \dfrac{s^2}{r^2}}$

72. $\dfrac{\dfrac{x+h}{x+h+1} - \dfrac{x}{x+1}}{h}$

73. $\dfrac{x^2-4}{x^2-4x+4} \cdot \dfrac{6-3x}{x+2}$

74. $\dfrac{9x^2-4}{\frac{9}{2}x^2-3x+2} \cdot \dfrac{9x^4-6x^3+4x^2}{3x^3-5x^2+2x}$

75. $\dfrac{4x^2-16}{x^2+3x-10} \div \dfrac{2x^2+10x+12}{x^2+8x+15}$

76. $\dfrac{x^3-25x}{4x^2} \cdot \dfrac{2x^2-2}{x^2-6x+5} \div \dfrac{\frac{1}{5}x^2+x}{7x+7}$

77. $\dfrac{(x+h)^2 - 3(x+h) - (x^2-3x)}{h}$

78. $\dfrac{x^{-1} - (x+2)^{-1}}{2}$

79. $(3x+2)^{\frac{1}{3}}(2)(4x-5)(4) + (4x-5)^2(\frac{1}{3})(3x+2)^{-\frac{2}{3}}(3)$

80. $(3x+1)^6(\frac{1}{2})(2x-5)^{-\frac{1}{2}}(2) + 6(3x+1)^5(3)(2x-5)^{\frac{1}{2}}$

81. $(x^2+9)^4(-\frac{1}{3})(x+6)^{-\frac{4}{3}} + 4(x^2+9)^3(2x)(x+6)^{-\frac{1}{3}}$

82. $\dfrac{(2x)(x^2-1)^4 - x^2(4)(x^2-1)^3(2x)}{[(x^2-1)^4]^2}$

83. $\dfrac{3x^2(x^2-5)^4 - x^3(4)(x^2-5)^3(2x)}{[(x^2-5)^4]^2}$

84. $\dfrac{3(x^2+4)^{\frac{1}{3}} - 3x(\frac{1}{3})(x^2+4)^{-\frac{2}{3}}(2x)}{[(x^2+4)^{\frac{1}{3}}]^2}$

85. $\dfrac{2x(1-x^2)^{\frac{1}{2}} - x^2(\frac{1}{2})(1-x^2)^{-\frac{1}{2}}(-2x)}{[(1-x^2)^{\frac{1}{2}}]^2}$

86. $\dfrac{(\frac{1}{3})(2x+3)^{-\frac{2}{3}}(2)(3x+2)^{\frac{1}{2}} - (2x+3)^{\frac{1}{3}}(\frac{1}{2})(3x+2)^{-\frac{1}{2}}(3)}{[(3x+2)^{\frac{1}{2}}]^2}$

1.1.5 Rationalize the Numerator (and completely simplify).

87. $\dfrac{\sqrt{x} - \sqrt{y}}{x^2 - y^2}$

88. $\dfrac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h}$

1.1.6 Solve the Equation.

89. $x^2 - x - 12 = 0$

90. $3x^2 + 5x = 12$

91. $2x - 4 = 2(x - 3)$

92. $3x + 2(x - 10) = 5(x - 4)$

93. $x^2 - 6x + 11 = 2$

94. $(x + 2)(x + 4) = 3$

95. $3x^3 = 15x^2 + 18x$

96. $x^3 + 84x = 19x^2$

97. $x^2 - 4x - 3 = 0$

98. $2x^2 + 8x + 3 = 0$

99. $2x^2 - 4x + 3 = 0$

100. $(x + 1)(\frac{2}{5}x - 1) = \frac{2}{5}$

101. $(\frac{1}{4}x - \frac{1}{2})(3x + 3) = 3$

102. $(\frac{2}{3}x - 1)(\frac{1}{2}x + \frac{1}{2}) = \frac{1}{2}$

103. $z^2(z + 2) - 4(z + 2) = 0$

104. $x^4 + x^3 - 4x^2 - 4x = 0$

105. $\dfrac{x + 1}{4} = \dfrac{1}{6} + \dfrac{2 - x}{3}$

106. $\dfrac{3x}{5} - \dfrac{x - 3}{2} = \dfrac{x + 2}{3}$

107. $\dfrac{3}{x + 4} - 7 = \dfrac{-4}{x + 4}$

108. $\dfrac{15}{6t^2 - t - 1} + \dfrac{3}{2t - 1} = \dfrac{2}{3t + 1}$

109. $\dfrac{-3}{x + 4} + \dfrac{7}{x - 4} = \dfrac{-5x + 4}{x^2 - 16}$

110. $S = \dfrac{p}{q + p(1 - q)}$ for q

111. $\dfrac{1}{f} = \dfrac{1}{p} + \dfrac{1}{q}$ for q

112. $\dfrac{1}{R} = +\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$ for R_2

113. $\sqrt{20 - 8x} = x$

114. $\sqrt{3 - x} - x = 3$

115. $x = 3 + \sqrt{5x - 9}$

116. $1 + \sqrt{6x + 1} = x$

117. $x^3 - 17x + 4 = 0$ given that 4 is a root

118. $3x^3 + 7x^2 - 22x - 8 = 0$ given that $-\frac{1}{3}$ is a root

1.1.7 Solve the Absolute Value Inequality - write the solution as an interval.

119. $|3x + 5| < 17$

120. $\left|\frac{3(x-1)}{4}\right| < 6$

121. $\left|3 - \frac{3}{4}x\right| > 9$

122. $1 < \left|x - \frac{11}{3}\right| + \frac{7}{3}$

123. $-\left|2 - \frac{x}{2}\right| + 1 \geq -1$

124. $-4|x + 2| + 5 \leq -7$

1.1.8 Polynomial Division

125. $(x^4 - 10x^2 - 2x + 3) \div (x + 3)$

126. $(x^3 - 2) \div (x - 1)$

127. $(-13x^3 + 10x^4 + 8x - 7x^2 + 4) \div (3 - 2x)$

128. $(x^4 + 6x^3 + 6x^2 - 10x - 3) \div (2x - 3)$

129. $\dfrac{x^4 + 6x^3 + 6x^2 - 10x - 3}{x^2 + 2x - 3}$

130. $\dfrac{2x^3 + 2x^2 - 2x - 15}{2x^2 + 4x + 5}$

131. $\dfrac{2x^5 - 8x^4 + 2x^3 + x^2}{2x^3 + 1}$

132. $\dfrac{4x^4 + 6x^3 + 3x - 1}{2x^2 + 1}$

1.1.9 Function Notation - evaluate the function as indicated, and simplify.

133. $f(x) = 3 - 7x$

   (a) $f(-1)$ (b) $f(\frac{1}{2})$ (c) $f(t) - f(-2)$ (d) $f(x+1)$

134. $f(x) = \dfrac{3x}{x-5}$

   (a) $f(0)$ (b) $f(\frac{5}{3})$ (c) $f(2) - f(-1)$ (d) $f(x+4)$

135. $g(x) = \dfrac{x+2}{x+1}$

   (a) $g(0)$ (b) $g(-1)$ (c) $g(3) + g(-5)$ (d) $f(x-2)$

136. $f(x) = \begin{cases} x + 8, & x < 0 \\ 10 - 2x, & x \geq 0 \end{cases}$

   (a) $f(4)$ (b) $f(-10)$ (c) $f(0)$ (d) $f(6) - f(-2)$

137. $f(x) = \begin{cases} 12, & x < -2 \\ 5x - 4, & -2 \leq x < 2 \\ 10x, & x \geq 2 \end{cases}$

   (a) $f(-2)$ (b) $f(2)$ (c) $f(1) + f(3)$ (d) $f(-5) + f(-4)$

138. $f(x) = 2x^2 + 5$

   (a) $f(x+2) - f(2)$ (b) $f(x+1) - f(x)$ (c) $\dfrac{f(x-3) - f(3)}{x}$ (d) $\dfrac{f(x+h) - f(x)}{h}$

§ In exercises 139-142, find and simplify the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ for each function.

139. $f(x) = 3x^2 + x + 5$ 140. $f(x) = -x^2 - 3x + 1$ 141. $f(x) = \dfrac{1}{2x^2}$ 142. $f(x) = \sqrt{x - 1}$

1.1.10 Function Operations

§ In exercises 143-146, below find

   (a) $(f \circ g)(x)$ (b) $(g \circ f)(x)$ (c) $(f \circ g)(3)$

143. $f(x) = x^2 + 3, \; g(x) = 4x - 1$ 144. $f(x) = \sqrt{x}, \; g(x) = x + 1$

145. $f(x) = 6x - 3, \; g(x) = \dfrac{x+3}{6}$ 146. $f(x) = 5x - 2, \; g(x) = -x^2 + 4x - 1$

§ In exercises 147-150,
(A) Find an equation for f^{-1}, the inverse function.
(B) Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.

147. $f(x) = 4x - 3$ 148. $f(x) = 8x^3 + 1$ 149. $f(x) = \dfrac{2}{x} + 5$ 150. $f(x) = \dfrac{2x - 3}{x + 1}$

§ Solve each inequality in exercises 151-154, and express each solution set in interval notation.

151. $\dfrac{3x + 5}{6 - 2x} \geq 0$ 152. $\dfrac{1}{x + 1} \geq \dfrac{2}{x - 1}$ 153. $\dfrac{x^2 - 3x + 2}{x^2 - 2x - 3} \geq 0$ 154. $\dfrac{x^4 - 3x^2 - 4}{x^2 - x - 20} < 0$