MATH 400 TOPICS IN MATHEMATICS

GRAPH THEORY

SPRING 2022 MTWF 10:00-10:50 HIRT 103

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Office Hours: Monday 2 - 3 pm
Tuesday 9 - 10:30 am and 1 - 2:30 pm
Wednesday 2 - 3 pm
Friday 1 - 2 pm
and by appointment

Course Description

The course will cover fundamental concepts related to graphs and trees. Specific topics will include cycles and paths, graph coloring, counting strategies, matchings, matrices and algorithms in graph theory, planar graphs, and modeling with graphs. We will also consider a variety of applications to active research areas, including computer science, data science, biology, engineering, and more.

Course Objectives

On successful completion of the course, students will:

- achieve command of the fundamental definitions and concepts of graph theory.
- understand and apply the core theorems and algorithms of graph theory, generating examples as needed.
- achieve proficiency in basic graph theory proof techniques such as bijections and minimal counterexamples.
- work on clearly expressing mathematical statements and arguments, both verbally and in writing.
- become familiar with the major viewpoints and goals of graph theory.
- apply graph theory based tools in solving practical problems.

Important Spring 2022 Dates

January	**19**	First Class Meeting (First Week Remote Meetings)
	21	Last Day to Add/Drop
March	**7-13**	Spring Break (No Class)
	25	Project Abstracts Due
	29	Advising Day (No Class)
April	**8**	Last Day to Declare Pass/Fail
	14-18	Easter Break (No Class)
	19	Last Day to Withdraw
	25	Project Paper Due
	29	Project Talks
May	**2-6**	Project Talks

Course Components

Homework

You will have several assignment due throughout the semester (generally one or two per unit). You should expect to spend a fair amount of time on each assignment - don't wait until the night before it's due to get started! You are free to work together on your assignments, but everyone must submit their own work, in their own words. If you need an extension on an assignment, please let me know ahead of the due date so the same extension can be offered to the rest of the class. Your lowest assignment grade (including a missed assignment) will be dropped when calculating your final grade.

Project

Homework assignments give you a chance to practice using the essential definitions and theorems of graph theory, but this is a subject that begs to be explored further and applied in a way you find interesting. At the end of the semester, our class will be hosting a mathematical conference, and you'll be an invited speaker.

Call for Abstracts *5% of Project Grade*

Participation in the conference requires submission of an abstract. This is a one to two paragraph overview of your chosen topic, and must be approved by the moderator. Your abstract should also include a list of at least two peer-reviewed sources (textbooks and accepted journal articles - *not* websites). You are free to choose any topic of interest to you that is related to graph theory. Your abstract **must be typed and emailed to me** by the deadline, so they can be compiled into a conference program.

Conference Proceedings *60% of Project Grade*

You will be expected to submit a brief article on your chosen topic to the conference proceedings. There is no set length on this article, as some of the most influential papers in mathematics have been only a few pages, while others have hundreds. However, you might want to aim for 2-4 pages in length. This article should provide the details of what you've uncovered in your research, with a clear thesis and using standard mathematical language.

Talks *35% of Project Grade*

You will have 15 minutes allotted for your talk. This time includes set up (getting slides ready on the projector, distributing handouts, etc) and time for questions from the conference attendees. Your actual talk should be about 10-12 minutes. Extra materials (such as slides and handouts) should be used when appropriate. Your talk should include a list of peer-reviewed sources used, and suggestions for further reading. You should not aim to cover every detail of your paper in these 15 minutes. Rather, focus on two or three main points. The talk should be directed at your classmates, with the more advanced or complicated explanations left in the paper itself.

Feedback and Awards

All conference attendees will have an opportunity to provide feedback on your talk; a sample comment form is included in this syllabus. Attendees will also score your talk, though these scores will remain confidential and will not be revealed to the speaker. The speaker with the highest total score will receive a small graph theory related prize. The runner up will receive an even smaller graph theory themed prize. Comments and scores will not influence your project grade.

Grades

Your final grade will be calculated as follows:

Component	Total Value
Homework (Lowest Dropped)	**60%**
Project	**40%**

Mathematics Department Grade Scale:

Grade	F	D	D+	C	C+	B	B+	A
Percentage	0-59	60-69	67-69	70-76	77-79	80-86	87-89	90-100

ADA and Learning Differences

Mercyhurst University is committed to making reasonable accommodations for qualified students, and employees with disabilities as required by law. Please refer to the HUB

https://lakersmercyhurst.sharepoint.com/sites/StudentsHub

and select the Services tab, then ADA Accommodations from the dropdown for instructions to request an accommodation. You may also contact Susan Reddinger, ADA Coordinator, ADA@mercyhurst.edu, 814-824-2362, Egan Hall 200. For students with questions about Academic Support, please refer to the HUB

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and select the Academic Resources tab, then Academic Support for more information.

Title IX Information

Mercyhurst is committed to providing an environment free from sex discrimination, including sexual harassment and sexual violence. Please refer to the HUB:

https://lakersmercyhurst.sharepoint.com/sites/StudentsHub

and select the Resources tab, then Title IX – Sexual Respect from the dropdown for more information. If you would like to file a sexual misconduct complaint, please contact Ann Miller, Title IX Coordinator and Compliance Officer, titleix@mercyhurst.edu, 814-824-2363. Please be aware that in compliance with Title IX, educators must report incidents of sexual assault/harassment, stalking, and domestic/dating violence. If you disclose any of these situations in class, in papers, or to me personally, I am required to report it to the Title IX Coordinator (or any of the Deputy Title IX Coordinators).

Course Evaluations

Near the end of the semester, you will be asked to complete an online course evaluation. The evaluation will be completed in class during the last two weeks of the semester using any laptop, tablet, or mobile device. The response tool allows you to note aspects of the course that helped you learn, as well as aspects that might be modified to help future students learn more effectively. You will receive an email letting you know when the evaluation window for our class is open. Please note that these course evaluations are anonymous and instructors do not see the results until after the grades for the course are submitted.

Covid

First Week
Our first week of classes for the Spring 2022 semester will be held remotely. Links to join the Zoom meeting will be posted on Blackboard. Additional changes to our schedule may be made according to updated University policies.

Masks
University policy requires all individuals to wear face coverings while indoors on campus. Masks are not required while sitting alone at your office desk or while eating.

Food and Drink in the Classroom
In light of the COVID-19 situation, eating is not permitted in classrooms, labs, or other academic spaces. A water bottle or cup with a lid (and preferably a straw) is permitted to be used in classrooms and labs to help prevent a student from becoming dehydrated. Masks should be pulled only slightly away from the bottom of the face to take a quick drink and immediately replaced to cover the mouth and nose.

Math 400 Spring 2022 Course Topics

Unit 0: Prerequisites

A review/introduction to some important definitions from other areas of mathematics, including sets and their operations, relations and functions, counting formulas, and matrices.

Unit 1: Fundamental Concepts

The historical development of graph theory, essential definitions related to graphs, and an overview of the types of graphs and related problems we'll study.

Unit 2: Degree, Paths, and Cycles

A closer look at how we travel on graphs, and how to detect if a route meeting certain criteria is possible. These types of problems are among the most common reasons for studing graph theory.

Unit 3: Connectivity and Subgraphs

We'll take a deeper look at connected graphs, split large graphs into smaller pieces, establish new ways of classifying graphs, and consider graphs as a solution in a variety of network flow problems.

Unit 4: Simple and Bipartite Graphs

Just because they're simple doesn't mean they can't be complicated. We'll look at the properties of these essential graphs in light of our new definitions.

Unit 5: Matrices

Matrices are used in linear algebra to simplify complex systems of equations. Likewise, they can be used in graph theory to simplify complex graph properties and help us find patterns, including those related to adjacency and optimization. We'll study a few types of matrices and some important applications of graph matrices.

Unit 6: Directed Graphs

Directed graphs have their own versions of many of the definitions we've already seen. We'll see how things change when we have a few directed edges in our graphs.

Unit 7: Trees

They look like simple graphs, are technically directed graphs, and are responsible for almost every advertisement you've seen on your phone or computer in the last few years. Trees are quickly becoming one of the most important types of graphs - in data science, machine learning, biology, and even business.

Unit 8: Matchings and Factors

Pairing things from two sets is a common problem that is well suited for the graph theory approach. We'll defined types of matchings; view some important principles and algorithms behind matchings; define graph coverings.

Unit 9: Coloring

Some useful theorems on adding color to graphs, which can be applied in a variety of ways: to edges, to vertices, or in the spaces in between.

Unit 10: Additional Topics

We'll look at some extra topics as time permits. Material will also be covered by request, so if there was something you were hoping to see that didn't make it into the previous units, let me know!